

Solution Sheet on Problem Set 4

**Fixed Income**

Deadline: 16.12.2021

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| **Task** |  | **Points Earned** |
| **Forwards to Interest**  a)  Different types of interest rates  (9 points) | The effective interest rate (also called yield-to-maturity ytm) is the total rate of return that will have been earned by a bond when it makes all interest payments and repays the original principal.  The spot rate is the rate of return earned by a bond when it is bought and sold on the secondaty market without collecting interest payments  In our plot, the effective interest rate and the continuous compounded interest rate are very similar, this is normal for very small values. The spot interest rate is similar to the other two for very small values (approx. for 0 – 1%) but deviates, as the interest rates increase. One reason for this behavior might be, that the spot interest rate is not applied to accumulated interest rates, but only to the initial capital. The YTM, as well as the continuous compounded interest rate take these into account. |  |
| b)  Z-Bond prices  (7 points) | The price of the zero coupon bond decreases with maturity due to the interest rate, until the maturity of 21.5, where the minimum is reached. Since the interest rate decreases with maturities > 14, the price of the bond is starting to increase up to maturity 37, when it’s deacreasing again. The price in the plot is dented as the facevalue. |  |
| c)  Forward Rate (3y to 20y) (4 points) | Based on Formula:    From the lecture notes, the Forward rate of a contract entering in 3 years and selling in 20 years is: 0.015 |  |
| **Fix.Inc. Calculus**  a)  Coupon bond price (4 points) | Based on Formula:    The price of the bond is: 115.95 |  |
| b) Yield-to-maturity  (4 points) | Based on the approximation formula for the YTM:    The YTM is: 0.00557 (0.56%) |  |
| c) Bond Duration’s  (6 points) | Based on the Formular (Dollar Dur, Adjusted Dur and Macaulay Dur): |  |
| d) Scenarios: description (6 points) | **Adjusted Yield Curves:**    From the graphs above, we can observe that adding a constant to the yield function (+/-0.005) leads to a parallel upward/downward shift of the yield curve, which is in accordance with the underlying mathematical and geometric properties of a function. Additionally, multiplying the yield by a constant >/< 1 leads to a squeeze/stretch of the yield curve, as stated by the yellow graph (\*1.3). Hence, (almost) all yields across the maturities are higher, whereby the tails are converging to the original yields and the yield at 15-year maturity has the highest difference to the original yield. Lastly, the transformation by stretching combined with a dynamic/dependent upward shift based on the maturity (\*0.3+(maturity/4000)) leads to a transformation that has (strictly) increasing yields over the maturities. |  |
| e) Scenarios: YTM (10 points) | |  |  |  | | --- | --- | --- | |  | **Price** | **YTM** | | **Spot Rate** (Y’=Y+0.005) | 98.00151 | 0.010606 | | **Spot Rate** (Y’=Y-0.005) | 137.725946 | 0.000478 | | **Spot Rate** (Y’=Y\*1.3) | 109.886897 | 0.007174 | | **Spot Rate** (Y’=Y\*0.3+tau/4000) | 96.258536 | 0.011144 | |  |  |  | |  |
| f) Scenarios: prices (10 points) | **Prices vs. Approximated Prices:**    From the graph above, we can conclude that the price approximation using the dollar duration and the absolute change in the yield-to-maturity (YTM) leads to better results in the increasing YTM. For lower YTMs (close to zero), the approximation shows quite a large difference to the real price, whereas the approximation with a YTM >0.01 leads to very small approximation differences. |  |
| **3.**  **Yield Curve** a) Yield curve patterns (4 points) | **Yield Curve:**    From the yield curve, we can observe that the interest rates on government bonds for the maturities of one and three years break the theoretical concave structure. For this to hold, we would assume higher interest rates for these maturities and hence absolute increasing but marginally decreasing interest rates. Furthermore, due to the inflation-targeting low interest rate policy, i.e. in Europe and in the US, the yield curve is fully defined at negative interest rates for maturities up to 20 years. Lastly, for the short maturities of 1m, 2m and 3m, we would also assume slightly increasing interest rates, whereby equal rates are observed in the data. |  |
| b)  Z-bond prices (6 points) | **Zero Coupon Bond Term Structure:** |  |
| c) Yield curve fit (8 points) | For the below regression:  x1 = m, i.e. the tenor  x2 = m^2, i.e. the tenor squared  x3 = m^3, i.e. the tenor cubed  A screenshot of a computer  Description automatically generated with medium confidence  Adding the fitted line to the chart from 3b)  **Chart, line chart  Description automatically generated** |  |
| d) 7 year z-bond (5 points) | Value of a 7y Zero Coupon Bond: 103.39362 (assuming Face value = 100) |  |
| e) Semi-annual coupon bond (5 points) | Value of a 3y semi annual Coupon Bond: 105.32263 (assuming Face value = 100) |  |
| f) Level, slope & curvature (6 points) | # level = y10  # slope = y10 -y3m  # curvature = (y2-y3m) - (y10 - y2)  It can be observed that the level (i.e. the 10y interest rate) steadily decreases from roughly 4% to below 0%.  Between 2000 and 2004 an increase in the slope can be seen. Given that we don’t see the same corresponding increase of the 10y rate a decrease in the 3month rate is implied. It can be theorized that this is a direct consequence of the dot com bubble. At the same time a decrease in curvature is examined. Due to the increase in the slope and the decrease in curvature it can be inferred that the 3 months rate dropped more relatively to the 2y and 10y rates.  Another very large move of the slope as well as curvature can been seen shortly after 2008 (financial crisis). The sharp increase in the slope must be from a sharp decrease in the 3 months rate, because we don’t observe a very large move in the level (10y rate). To soften the impact of the financial crisis the Central Bank decreased interest rates on the short end.  Interestingly, for the Covid crisis which started in March 2020 we do not see such large changes in level, slope, and curvature. One potential explanation for this could be that the Central Bank was already effectively at the zero lower bound at that point in time and from a rates perspective did not have much room to react to the crisis. Instead, the Central Bank had to resort to other measures such as other methods such as enormous asset purchase programs (e.g. PEPP for the ECB).  **Chart, line chart, histogram  Description automatically generated** |  |
| g) Different Spreads (6 points) | The plot below displays the spread of 10y AA corps as well as BBB corps over the 10y Government Rate. This can also be referred to as the risk premium, as generally the assumption is that the government risk in the respective currency is the risk free rate for this tenor. Clearly, the risk premium for the AA rated corps is lower than the BBB rated corps, which are barely IG rated (anything below BBB- is considered high yield). In addition to the higher risk premium, the lower rated corps also have a higher volatility. For both the AA and BBB corps an increase in risk premium can be observed at the start of the Covid crisis in March 2020, which subsequently recovered fairly fast – this is partly because of the central banks (e.g. Fed, ECB, SNB) that bought these bonds in the aftermath of the Covid crisis to ensure that the bond market remains sufficiently liquid and that companies could refinance themselves at cheap levels. **Chart, line chart  Description automatically generated** |  |